A Reformulation of the Relativistic Transformation Between Coordinate Time and Atomic Time

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In this report, the relativistic time transformation is reformulated to allow simpler time calculations relating analysis in a solar system frame (using coordinate time) with Earth-fixed observations (using atomic time). After an interpretation of terms, this simplified formulation is used to explain the conventions required in the synchronization of a world-wide clock network. In addition, two synchronization techniques—portable clocks and radio interferometry—are discussed in terms of the relativistic time transformation.

I. Introduction

In the relativistic analysis of the very long baseline interferometry (VLBI) and spacecraft radio metric data, primary calculations are often most conveniently made in a frame at rest with the solar system barycenter. However, observations in these applications are often made relative to an Earth-fixed frame. Consequently, such analyses usually involve a relativistic time transformation (Refs. 1, 2) between the solar system frame (using coordinate time) and Earth-fixed observers (using atomic time). In this report, the time transformation, including both speed and potential terms, is reformulated in order to facilitate both interpretation and analysis in these applications. After an interpretation of the terms in the reformulation, the transformation is used to consider the synchronization conventions associated with a world-wide clock network. Since clock stabilities are beginning to routinely enter a relativisticly significant range (10^{-12} to 10^{-13}), a discussion of such conventions is presently more than an academic exercise. Finally, two synchronization techniques, portable clocks and VLBI, are analyzed in terms of the simplified time transformation.

II. Time Transformation Reformulation

In many VLBI and spacecraft applications, relativistic effects are most conveniently handled by performing primary calculations in a frame at rest with respect to the solar system barycenter and then making a relativistic transformation to Earth-fixed antenna observers. Consequently, these calculations usually involve a relativistic time transformation from the solar system frame to Earth-fixed observers. In this section, the time transformation is manipulated in a simple manner to cast it in a form that is more convenient for most applications.

In this analysis of the time transformation, approximations will be guided by the following considerations. State-of-the-art oscillator technology (H-maser standards) can, at best, provide clocks with a long-term stability of the order of $\Delta f/f = 10^{-14}$. Because of this instrumental limitation on time measurement, theoretical rate corrections ($d\tau/dt$) of the order of 10^{-15} or less are presently not experimentally significant. Consequently, terms that lead to clock rate corrections of the order of 10^{-15} or less will not be retained in the following analysis.

Suppose that observers in the solar system frame note that a given event occurs at coordinate time t at a given point j on the Earth's surface. Earth-fixed observers at this point will note that the same event occurs at proper time τ_j according to their (atomic) clock. Figure 1 defines the location of point j as measured in the solar system frame. Note that the position vector of point j ($\mathbf{Y}_j$) is represented as the sum of a vector to the Earth center of mass ($\mathbf{X}_e$) and a vector ($\mathbf{X}_j$) from the Earth's center of mass to the given point. This separation of orbit and spin geometry leads naturally to a simplified version of the transformation from coordinate time t to proper time τ_j, as the following manipulations will show.

If one retains only the most significant terms in the metric tensor (i.e., the terms that lead to rate corrections greater than 10^{-15}), then the relativistic transformation (Ref. 1) relating times in the two frames is given by

$$\frac{d\tau_j}{dt} = \left[1 - \frac{(\dot{\mathbf{Y}}_j)^2}{c^2} + \frac{2\phi(\mathbf{Y}_j)}{c^2}\right]^{1/2} \tag{1}$$

where

$$\dot{\mathbf{Y}}_j = \mathbf{V}_e + \mathbf{V}_j$$

and $\phi(\mathbf{Y}_j)$ is the Newtonian potential at point $\mathbf{Y}_j$ in the solar system frame. The order of magnitude of the various terms in Eq. (1) is as follows:

$\frac{V_e}{c} \approx 10^{-4}$ for Earth orbital speed

$\frac{V_j}{c} \approx 10^{-6}$ for Earth observer rotational speed with respect to Earth's center of mass

$\frac{\phi}{c^2} \approx 10^{-8}$ for gravitational potential at Earth's orbit

Therefore, to order 10^{-15} in the rate expression, we have

$$\frac{d\tau_j}{dt} = 1 - \frac{V_e^2 + V_j^2 + 2\mathbf{V}_e \cdot \mathbf{V}_j}{2c^2} + \frac{\phi}{c^2} \tag{2}$$

The last equation must be integrated to give proper time as a function of coordinate time, which yields

$$\tau_j(t) = \tau_c + \tau_j^I + t - t_c - \int_{t_c}^{t} \left[\frac{V_e^2 + V_j^2 + 2\mathbf{V}_e \cdot \mathbf{V}_j - 2\phi}{2c^2}\right] dt \tag{3}$$

where the initial value for $\tau_j(t)$ has been separated into two parts, τ_c and τ_j^I. The term τ_c is common to all clocks while the term τ_j^I allows for possible adopted differences in initial clock readings at $t = t_c$. These terms will be discussed in Section III.

Two terms, $\mathbf{V}_e \cdot \mathbf{V}_j$ and ϕ, will now be manipulated into more useful forms. As shown below, these manipulations lead to a time transformation for Earth-fixed clocks that does not involve an integral over $\mathbf{X}_j$, the clock's motion relative to the Earth's center of mass.

The potential term ϕ can be separated into a sum of two potential terms:

$$\phi(\mathbf{Y}_j) = \phi_e(\mathbf{X}_j) + \phi_R(\mathbf{X}_e + \mathbf{X}_j) \tag{4}$$

where $\phi_e \approx 10^{-9}\, c^2$ is the Earth potential, and ϕ_R is due to all other bodies. The Earth potential is very nearly constant for a given Earth-fixed point, while the potential ϕ_R varies as the clock moves about due to both Earth spin and Earth orbital motion. In order to separate the Earth spin and Earth orbital motion, expand ϕ_R as follows:

$$\phi_R(\mathbf{X}_e + \mathbf{X}_j) \approx \phi_R(\mathbf{X}_e) + \nabla\phi_R(\mathbf{X}_e) \cdot \mathbf{X}_j \tag{5}$$

It is readily shown that quadratic terms in this expansion are of the order of 10^{-17}. If one neglects relativistic terms of the order 10^{-15}, the gradient $\nabla\phi_R$ is the acceleration of Earth's center of mass so that

$$\phi_R(\mathbf{X}_e + \mathbf{X}_j) = \phi_R(\mathbf{X}_e) - \mathbf{a}_e \cdot \mathbf{X}_j \tag{6}$$

The time transformation can be further modified by integrating the $\mathbf{V}_e \cdot \mathbf{V}_j$ term by parts to obtain

$$\int_{t_e}^{t} \mathbf{V}_e \cdot \mathbf{V}_j \, dt = \mathbf{V}_e(t) \cdot \mathbf{X}_j(t) - \mathbf{V}_e(t_e) \cdot \mathbf{X}_j(t_e) - \int_{t_e}^{t} \mathbf{a}_e \cdot \mathbf{X}_j \, dt \tag{7}$$

Substituting Eqs. (4), (6) and (7) in Eq. (3), we obtain the expression

$$\tau_j(t) = t - t_e - \int_{t_e}^{t} \left[\frac{V_e^2 - 2\phi_R(\mathbf{X}_e) + V_j^2 - 2\phi_e(\mathbf{X}_j)}{2c^2} \right] dt - \frac{\mathbf{V}_e(t) \cdot \mathbf{X}_j(t)}{c^2} + \frac{\mathbf{V}_e(t_e) \cdot \mathbf{X}_j(t_e)}{c^2} + \tau_j^I + \tau_e \tag{8}$$

Note that the two acceleration integral terms produced by the velocity and potential terms have canceled. Furthermore, except for the $\mathbf{V}_e \cdot \mathbf{X}_j$ terms, the orbital and spin motions have been separated.

For a clock fixed with respect to Earth, the speed V_j and potential $\phi_e(\mathbf{X}_j)$ are constant to about one part in 10^6. Therefore, to excellent approximation, we obtain for an Earth-fixed clock

$$\tau_j(t) = t - t_e - \int_{t_e}^{t} \left[\frac{V_e^2 - 2\phi_R(\mathbf{X}_e)}{2c^2} \right] dt - \frac{\mathbf{V}_e(t) \cdot \mathbf{X}_j(t)}{c^2} + \frac{\mathbf{V}_e(t_e) \cdot \mathbf{X}_j(t_e)}{c^2} - \left[\frac{V_j^2 - 2\phi_e(\mathbf{X}_j)}{2c^2} \right] (t - t_e) + \tau_j^I + \tau_e \tag{9}$$

These expressions for proper time simplify the analysis of clock synchronization which follows.

III. Clock Synchronization Conventions

World-wide timekeeping is now accomplished by a network of atomic clocks placed at various locations over the Earth. In this network, member clocks are periodically synchronized with a master clock, which is carefully maintained at a fixed location. ("Master clock" in practice is the average time reading of a set of reference atomic clocks. Specific techniques for synchronization will be discussed in Section V on the basis of the relativistic transformation.) Present synchronization work, based on the principles of classical physics, assumes that clocks, once synchronized in time and rate, will continue to indicate the same times, within instrumental accuracy, wherever they are moved on Earth's surface. However, relativistic analysis, such as Eq. (8), indicates that classical assumptions may not be adequate if clock accuracies surpass the μs level in time and the 10^{-12} level in rate. That is, sufficiently accurate clocks can lose synchronization due to relativistic effects if they are separated on the Earth's surface. Consequently, an accurate clock network based on relativity theory must take these effects into account.

An understanding of the synchronization problem is facilitated by the relativistic formulation in Eq. (9) which connects coordinate time with the proper time of a given Earth-fixed clock. Even though this equation is not a direct comparison of Earth-fixed clocks, it contains all the information needed to study the synchronization problem, provided the various terms are properly interpreted. The following discussion attempts such an interpretation with emphasis on the establishment of synchronization conventions. Even though some aspects of this discussion are relatively well-known, they have been included, sometimes without reference, for the sake of completeness.

First, we will divide the terms of the time transformation Eq. (9) into two categories, terms that are the same for all clocks and terms that are different:

$$\tau_j(t) = t - t_e + \Delta t_s + \Delta t_j + \tau_e \tag{10}$$

where the common terms are given by

$$\Delta t_s \equiv - \int_{t_e}^{t} \frac{V_e^2 - 2\phi_R(\mathbf{X}_e)}{2c^2} \, dt \tag{11}$$

and the clock-specific terms by

$$\Delta t_j \equiv - \left[\frac{V_j^2 - 2\phi_e(\mathbf{X}_j)}{2c^2} \right] (t - t_e) - \frac{\mathbf{V}_e(t) \cdot \mathbf{X}_j(t)}{c^2} + \frac{\mathbf{V}_e(t_e) \cdot \mathbf{X}_j(t_e)}{c^2} + \tau_j^I \tag{12}$$

The common term Δt_s contains the factors that cause the same rate offset for all clocks: the speed of the Earth center-of-mass and the "clock-invariant part" of the potential which is located at the Earth center-of-mass. Since this term is common to all clocks in the network, it will not cause a loss of synchronization. That is, this term is not significant in "Earth-bound" comparisons of clocks but is significant in transformations from Earth-fixed clocks to coordinate time. In practice, the common term must be modified to account for any conventions affecting overall clock rates. For example, in principle, the present system defines the second so that all clocks run at the same average rate as coordinate time (Ref. 1). This rate definition is represented formally in Δt_s by subtracting the time-average rate from the total rate in Eq. (11) as follows.

$$\widetilde{\Delta t}_s \equiv \Delta t_s - \overline{\Delta t}_s = -\int_{t_c}^{t} \frac{\Delta V_e^2 - 2\Delta\phi_R(\mathbf{X}_e)}{2c^2}\, dt \qquad (13)$$

where

$$\Delta V_e^2 = V_e^2 - \overline{V_e^2}$$

$$\Delta\phi_R = \phi_R - \overline{\phi_R}$$

This rate adjustment, which is of the order of 10^{-8}, leaves only the periodic effects in Δt_s. Even though these periodic effects do not cause loss of synchronization, they must still be included in transformations between proper time and coordinate time. For example, the predominant effect, orbital eccentricity, has an integrated amplitude of approximately 2 ms and an annual period.

The clock-specific term Δt_j can lead to synchronization loss between Earth-fixed clocks. This term can be subdivided into three categories of time dependence: constant, linear, and periodic. The constant terms are defined by the synchronization convention established below.

In the second category, the linear term, $[v_j^2/2 - \phi_e]$ $(t - t_c)$, is a rate correction based on clock geopotential and speed relative to Earth's center of mass. Note that this term is essentially the effective potential at point $\mathbf{X}_j$ as seen in an Earth-fixed frame. That is, the gradient of $V_j^2/2 - \phi_e$ gives the sum of the "centrifugal force" and gravitational force at that point. Since mean sea level represents, to good approximation, a surface of constant effective potential, clocks at sea level should run at essentially the same rate without relativistic corrections. However, for two arbitrary Earth-fixed clocks, the differential rate correction is easily calculated on the basis of differential altitude by the approximate formula $g\Delta h$, which predicts that the rate correction changes by approximately 1.1×10^{-13} per kilometer of altitude above mean sea level. For airborne clocks, it is readily shown that differential rate corrections of the order of 10^{-12} are possible. (In the airborne case, of course, $V_j^2 - 2\phi_e$ is not necessarily a constant at a given altitude since the clock is no longer an Earth-fixed object).

Finally, in the third category, the periodic term $\mathbf{V}_e(t) \cdot \mathbf{X}_j(t)$ is never greater than 2 μs and is essentially diurnal since $\mathbf{V}_e$ changes very little over one day. This term is a relativistic consequence of the time transformation between frames and corresponds to the special relativity clock synchronization correction. That is, it accounts for the relativistic principle that simultaneous events in one frame are not necessarily simultaneous in a frame passing by with speed V_e. Consequently, it is of significance in transformations from Earth-fixed time to coordinate time but is not present in "Earth-bound" comparisons between Earth-bound clocks. This fact is supported analytically by noting that the periodic term "changes" to match another clock if the two clocks are brought together on Earth.

The following conventions regarding synchronization are designed to accommodate these clock-specific terms. Since the linear terms can lead to gross disagreements between clocks over long time periods, they will be removed, either explicitly or implicitly, by making appropriate location-dependent definitions of clock rate. In principle, these corrections could be applied by means of explicit on-site rate adjustments based on a fundamental physical process. For example, at each location a second could be established in terms of a particular altitude-dependent number of cycles (Ref. 3) on a cesium beam frequency standard where the cycle-count differential between altitudes would be based on the calculated differential in effective potential. Since these rate adjustments are of the order of 10^{-13}, the oscillators would necessarily have to be capable of independent (absolute) calibration at a few parts in 10^{-14}. Unfortunately, routine calibrations at this level are not feasible at present. In practice, this rate adjustment will be implicitly applied in a differential sense whenever a worldwide clock network is kept in time synchronization. For example, as in the present system, a "master clock" would be utilized, at a given location, to define the second and maintain a reference time. Other clocks over the world would then be forced into synchronization by means of "Earth-bound" synchronization techniques (see Section V). Since the synchronization process prevents clock divergence, the appropriate differential rate correction will automatically be implicitly applied without recourse to relativistic calculations.

Since the periodic term $\mathbf{V}_e(t) \cdot \mathbf{X}_j(t)$ does not affect the synchronization of Earth-bound clocks, it is not of consequence in the establishment of a synchronization convention.

In order to complete the synchronization convention, the constant term τ_j^I must be defined. This term will be defined by requiring that the clocks exhibit zero disagreement *on the average* according to solar system observers. This goal is accomplished by letting

$$\tau_j^I = -\mathbf{V}_e(t_c)\cdot\mathbf{X}_j(t_c)/c^2 \tag{14}$$

As we shall see in Section V, this definition is appropriate for synchronizing clocks according to Earth-bound observers.

By applying the definitions and conventions described above, one obtains a standardized time transformation for clock j:

$$\tilde{\tau}_j(t) = t - t_c - \int_{t_c}^{t} \frac{\Delta V_e^2 - 2\Delta\phi_R(\mathbf{X}_e)}{2c^2}\,dt - \frac{\mathbf{V}_e(t)\cdot\mathbf{X}_j(t)}{c^2} + \tau_c \tag{15}$$

Note that the time transformation no longer involves an integral over clock coordinates but only over coordinates for the Earth's center-of-mass. Therefore, relative to the original transformation, time calculations are much simpler.

In summary, with the conventions outlined above, the network clocks would be given selected initial times (at coordinate time t_c) and the same average rate (i.e., $d\tau/dt = 1$) according to solar system observers. With these conventions, the clock network could be kept in synchronization according to Earth-bound observers by means of two synchronization methods now in use. These two techniques, portable clocks and VLBI, will be discussed in Section V in terms of these synchronization conventions.

IV. VLBI Time Delay

The VLBI time delay is readily calculated using Eq. (15) as follows. Suppose that radio waves emitted by a distant source are observed by two Earth-fixed antennas. Let a given wavefront reach antenna 1 at time t and antenna 2 at t' when observed in the solar system frame. According to the two antenna teams, the wavefront arrives at time $\tilde{\tau}_1(t)$ at antenna 1 and time $\tilde{\tau}_2(t')$ at antenna 2. When the two antenna teams compare arrival times, they will calculate the "geometric" delay:

$$\tau_g(t) \equiv \tilde{\tau}_2(t') - \tilde{\tau}_1(t) \tag{16}$$

We have assumed that the antenna clocks have been synchronized according to the conventions described in Section III.

Since $|t' - t|$ is less than 30 ms for Earth-fixed baselines, the terms containing t' can be expanded about t to yield

$$\tau_g(t) = \tilde{\tau}_2(t) - \tilde{\tau}_1(t) + \dot{\tilde{\tau}}_2(t)\,(t' - t) \tag{17}$$

$$= \left[1 - \frac{\Delta V_e^2 - 2\Delta\phi_R(\mathbf{X}_e)}{2c^2} - \frac{\mathbf{V}_e\cdot\mathbf{V}_2}{c^2}\right](t' - t) - \frac{\mathbf{V}_e(\mathbf{t})\cdot\mathbf{B}(t)}{c^2} \tag{18}$$

where the baseline $\mathbf{B}$ equals $\mathbf{X}_2 - \mathbf{X}_1$. In this expression, we have neglected a $\mathbf{a}_e\cdot\mathbf{x}$ term and terms of order higher than the first in $t' - t$ with negligible loss of accuracy. Note that the geometric delay is equal to the "coordinate time delay", $t' - t$, plus transformation corrections of two types. The first type is a "time dilation" correction, consisting of three terms proportional to $t' - t$. It is easily demonstrated that these terms are less than 0.5 cm in magnitude. Consequently, these corrections are of marginal importance for even the most ambitious VLBI applications.

The second correction category, which corresponds to the clock synchronization correction (or aberration correction) found in a special relativity treatment, can be estimated as follows:

$$\frac{\mathbf{V}_e\cdot\mathbf{B}}{c} \lessapprox 10^{-4} \times 6000\ \mathrm{km} = 600\ \mathrm{m} \tag{19}$$

Since $\mathbf{V}_e$ changes very little over a day, this term exhibits essentially diurnal time variations. In time delay calculations, this large correction must be treated very precisely.

Up to this point, the coordinate time delay $t' - t$ has been treated in a general fashion and could denote any two events recorded by relevant solar system observers. In VLBI applications, the times, t and t', denote the arrival times of a given wavefront at two antennas as seen by solar system observers. The description of this wavefront in VLBI applications can be divided into two formulations: a plane-wave description for sources at "infinite" distances (e.g., extragalactic sources) and a spherical wave description for "close" sources (e.g., a spacecraft in the solar system). Since this article is primarily concerned with the relativistic time transformation of a given coordinate time delay from solar system observers to antenna observers, a general discussion of delay calculations, including all factors, will not be attempted. However as an example, the time delay for an extragalactic source will be analyzed.

The time delay for an extragalactic source can be derived by first calculating the delay observed in the solar system frame and then transforming to antenna observers according to Eq. (18). We will give the signal a plane-wave representation that ignores transmission media and general relativity effects. According to solar system observers, the delay for a plane wave is easily shown to be given by

$$t' - t = -\frac{\mathbf{S}\cdot\mathbf{B}}{c\,[1 + \mathbf{S}\cdot(\mathbf{V}_e + \mathbf{V}_2)/c]} \tag{20}$$

where S is a unit vector in the direction of the radio source relative to the solar system barycenter. The observed time delay is obtained by inserting this expression into the time transformation, Eq. (18), to obtain

$$\tau_g(t) = -\left[1 - \frac{\Delta V_e^2 - 2\Delta\phi_R}{2c^2} - \frac{\mathbf{V}_e\cdot\mathbf{V}_2}{c^2}\right] \cdot \frac{\mathbf{S}\cdot\mathbf{B}}{c\,[1 + \mathbf{S}\cdot(\mathbf{V}_e + \mathbf{V}_2)/c]} - \frac{\mathbf{V}_e(t)\cdot\mathbf{B}(t)}{c^2} \tag{21}$$

All quantities in this expression are evaluated at time t, the time the wave front reaches antenna 1.

As an alternate approach, the geometric delay can easily be derived to order v/c on the basis of a geocentric approximation (Ref. 4). In that derivation, the $\mathbf{V}_e\cdot\mathbf{B}$ term enters the delay as a result of the aberration correction to the source direction. As indicated by the two derivations, this large term can be viewed in two ways. For Earth-bound observers, it is a *geometric* correction applied to the position of the source. For solar system observers, it is viewed as a *time* correction representing a loss of synchronization between Earth-fixed clocks.

The preceding analysis of the geometric delay will facilitate the discussion of VLBI clock synchronization that follows in the next section.

V. Clock Synchronization Techniques

This section will show how two synchronization techniques, portable clocks and VLBI, can be used to synchronize a world-wide clock network according to the synchronization conventions defined in Section III. The portable clock technique will be discussed first.

In the present world-wide timekeeping network, a set of atomic clocks ("the master clock") at one location is used to define a reference time. Clocks at other locations around the world are periodically resynchronized by comparing them with a portable clock that is carried to each member clock. Before traveling overseas each time, the portable clock is synchronized on-site with the master-clock. In this manner, a world-wide network of clocks is kept in synchronization at the level allowed by the instrumental and transportation stability of the clocks involved.

Let a portable clock be synchronized with the master clock at coordinate time, $t = t_0$. Then let the portable clock follow some path[1] $\mathbf{X}_p(t)$ over Earth to some member of the clock network. (Note that $\mathbf{X}_p(t)$ and $\mathbf{V}_p(t)$ consist of Earth-spin effects as well as clock transportation). After the portable clock has reached the member clock j at time t', the clock-specific correction for the portable clock will be

$$\Delta t_p = -\int_{t_0}^{t'} \frac{V_p^2(t) - V_m^2 - 2\,[\phi_e(\mathbf{X}_p) - \phi_e(\mathbf{X}_m)]}{2c^2}\,dt - \frac{\mathbf{V}_e(t')\cdot\mathbf{X}_j(t')}{c^2} \tag{22}$$

where V_p and V_m are the geocentric speeds of the portable and master clock. (We have not included the other terms in Eq. (10) in this discussion since they are common to all clocks and do not affect synchronization). The integral term in this expression accounts for the fact that the master clock rate adjustment (passed on to the portable clock during synchronization) will not suppress the $V_p^2 - 2\phi_e$ integral for the portable clock once it starts its journey and changes its geocentric position and speed.

According to the synchronization convention established in Section III, the portable clock-member clock comparison must be handled as follows. The desired value for the member clock is given by

$$\Delta t_j = -\frac{\mathbf{V}_e(t')\cdot\mathbf{X}_j(t')}{c^2} \tag{23}$$

[1]Relative to Earth center-of-mass.

Thus, comparing Eq. (22) and Eq. (23), we see that the portable clock must be corrected to account for the speed-potential integral that has accumulated in transit:

$$\Delta\tau_p = \Delta t_p - \Delta t_j$$

$$= -\int_{t_0}^{t'} \frac{V_p^2(t) - V_m^2 - 2\left[\phi_e(\mathbf{X}_p) - \phi_e(\mathbf{X}_m)\right]}{2c^2}\, dt \quad (24)$$

For one day transit times, this correction can be of the order of $10^{-12} \times 10^5$ s = 100 ns. Furthermore, the portable clock rate will differ from the conventional rate for site j by

$$\frac{V_j^2 - V_m^2 - \phi_e(\mathbf{X}_j) - \phi_e(\mathbf{X}_m)}{2c^2}$$

so that clock rate comparisons must include this correction factor. Thus, we see that, during transit, the periodic term $\mathbf{V}_e \cdot \mathbf{X}_p$ changes into the appropriate value while the linear term loses its adjustment and must be corrected.

It is interesting to note that the integral contained in Eq. (24) is essentially the theoretical time gain predicted by Hafele and Keating for their Earth-circumnavigation experiment (Ref. 5). In that paper, theoretical calculations only considered geocentric speed and geopotential effects. With a more general approach, the present formulation indicates that this integral is the total time gain, provided one can neglect rate terms less than 10^{-15}. Thus, the warning in Ref. 5 that effects of the sun and moon may not be entirely negligible appears to be unwarranted for present clock stabilities.

Clock synchronization by means of VLBI is conceptually, if not operationally, straightforward. For a given natural source, the time delay is measured between two antennas and appropriately corrected for transmission media and instrumental delays. The resulting delay should be equal to the geometric delay calculated according to Eq. (21). (We assume here that geophysical and astronomical parameters are known with sufficient accuracy.) Any difference between the measured delay and the calculated delay represents the synchronization loss between antenna clocks. In this manner, a world-wide system of clocks could be synchronized at interferometer accuracies.

VI. Experimental Tests

In a treatment of this nature, some discussion should be devoted to the tests of relativity that are suggested by the reformulation. As indicated in Section III, only the "effective potential" term will be evident in "Earth-bound" comparisons of Earth-bound clocks. Contingent on instrumental feasibility, several Earth-bound experiments might be suggested to test the presence of this effect. One experiment, involving airborne clocks (Ref. 5), has already been carried out. As an alternate approach, an experiment could be designed to take advantage of the clock synchronization precision of the VLBI technique. Time synchronization at the 10 ns level is now feasible with current VLBI instrumentation (Ref. 4). With this precision, a typical Earth-fixed rate differential of 10^{-13} (1 km altitude differential) would be visible in about three days. However, a test of this type requires "station-clock" rate stability and calibration at a few parts in 10^{-14}. Except perhaps at standards labs, this clock requirement would presently be the most difficult aspect of the VLBI approach.

VII. Summary

In the preceding sections, a reformulation of the relativistic time transformation has simplified interpretation of the various effects entering the transformation between coordinate time and Earth-bound proper time (atomic time). Based on this analysis, the conventions required for the synchronization of a world-wide clock network have been investigated. In addition, the new formulation has simplified a relativistic analysis of the "geometric" delay measured in VLBI applications. Finally, a brief discussion has been devoted to possible "Earth-bound" experimental tests of predictions of the theory.

References

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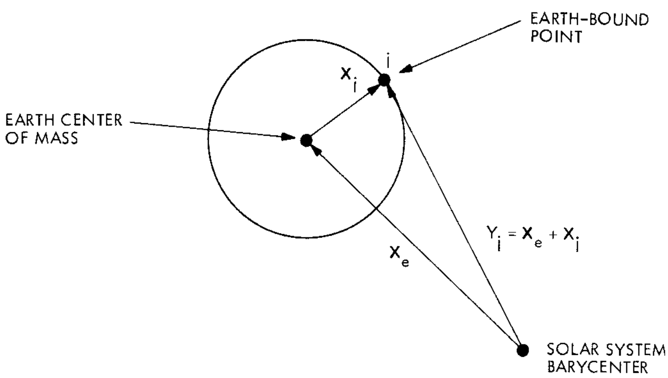

Fig. 1. The position vector of Earth-bound point *j* as measured by solar system observers